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Stock Options: An Example of Catastrophe Myopia?

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Stock Options: An Example of Catastrophe Myopia?

Summary.

1. On 19th October, 1987 (Black Monday) investors who had purchased "portfolio insurance" found that it did not work. The problem was that it involved an implicit assumption that share prices were "continuous".

2. Portfolio insurance is now purchased through the options market. The basic principle has not changed, but the risk takers have. The investors who lost out in 1987 were the owners of ordinary share portfolios, with capital equal to 100% of their exposure. Those at risk today include dealers, whose capital is a fraction of their exposure.

3. Dealers' models for option pricing seek to allow for price discontinuity. However, the extent to which prices are discontinuous is dependent on the size of the aggregate options exposure. Thus if discontinuity was allowed for, options prices would increase as the options exposure grows relative to the stock market. Increased discontinuity means an increased risk of sudden large price movements, which render markets more liable to self-feeding spirals.

4. Options should be priced in relation to the risk involved, including the increase in risk that accompanies an increase in the market's size. In practice they are not, as they simply respond to short term changes in actual volatility.

5. The risks being underwritten are only rarely unprofitable, but on such occasions the losses can be massive. This type of risk is usually underpriced and undercapitalised. Such habitual excessive confidence on the part of the insurers is know as “catastrophe myopia”.

6. The insurance of systemic risks, however priced, will produce occasional and extreme shocks. Catastrophe myopia leads to the underpricing of options and underpricing has probably contributed to the rapid growth in the options market. It is this growth in the size of the options market, rather than the pricing, which poses the real threat to the stock market and the economy.

7. Catastrophe myopia leads to undercapitalisation as well as underpricing. It is therefore probable that a number of financial institutions will become bankrupt in the next crash, thus adding to its severity.

1. Introduction.

In this report we show that the options market provides insurance to investors against the risk of a major sell off. Unlike life, fire or motor insurance, however, the aggregate risk is not significantly less than the sum of the individual risks, as it is systemic rather than specific. The reduction in risk by those taking out insurance must therefore be matched by the increased risk of those providing the insurance.

We estimate that the proportion of the US equity market which is effectively insured is probably approaching 10%. In the event of a 30% fall in the US market, matched by similar falls in other major markets, the total payout on this insurance would be of the order of $300-$400 bn. This compares with the combined equity of Merrill Lynch, Morgan Stanley, JP Morgan, Bankers Trust and Goldman Sachs of $33 bn. Thus if this risk was being underwritten by the option dealers, such a fall would be likely to cause widespread bankruptcy. Much of the risk, however, is being taken by the owners of investment portfolios. In stark contrast to other insurance markets, the options market does not decrease the total sum at risk, it merely shifts its incidence.

There are, however, other risks associated with the writing of options which, while they need not be assumed by any individual dealer, must be borne by dealers in aggregate. These risks arise from the volatility of the market rather than from rises or falls in its level. We show in this report that any major change in the level of the market will be highly costly to options dealers.

Of necessity, if the market falls sharply, dealers will be obliged to “rebalance” their portfolios by selling stocks, in order to reduce their exposure to further falls. This requirement to sell in a falling market will tend to increase the volatility of the market and render it liable to a self-reinforcing spiral. It is possible, but unlikely that such a spiral could set off the crash to which the stock market's extreme over-valuation has clearly rendered it vulnerable. What does seem probable, however, is that the size to which the options market has grown is such that it will tend to accentuate the scale of the crash, set off by other forces and increase the "price discontinuity" in the stock market, i.e. the extent to which large price movements will take place without transactions being possible at intermediate prices. "Circuit breaker" rules, which shut markets when prices move by more than a given amount, are more likely to intensify than limit the problems of price discontinuity. They do not limit dealers’ risks, they simply limit their ability to cover them.

2. Stock Options as Insurance.

Stock options can be used either for speculation or as a form of insurance. Investors in the stock market who wish to limit their exposure to market falls can, for example, buy a put option[1]. If their fears are realised, this will enable them to limit their losses. The more the market price of the underlying asset exceeds the exercise price of the option, the less will be their insurance cover and the lower will the price of the put option be.[2] Like all insurance, the cover provided has a cost. The equivalent of an insurance premium in the options market, is the price of the option.

Thus, as in any insurance market, the options market allows a risk averse investor to limit the extent of exposure to the equity market. Unlike a normal insurance market, however, the options market also allows risk taking investors to increase theirs. If a normal insurance market expands, this tends to reduce the impact of the underlying risk, through the “law of large numbers”, since most risks insured are specific, rather than systemic. In contrast, if the options market expands, the total amount of risk involved with stock market fluctuations rises by even more than the increase in the market's value. This increase in risk is analogous to that which accompanies an increase in debt. There can be no increase in net debt, as every borrower must have an equal lender, but the risk of default rises with the expansion of gross debt. It is equally wrong to assume that the expansion of options markets does not involve increased risk. The increased risk of the insurers equals the reduced risk of the insured.

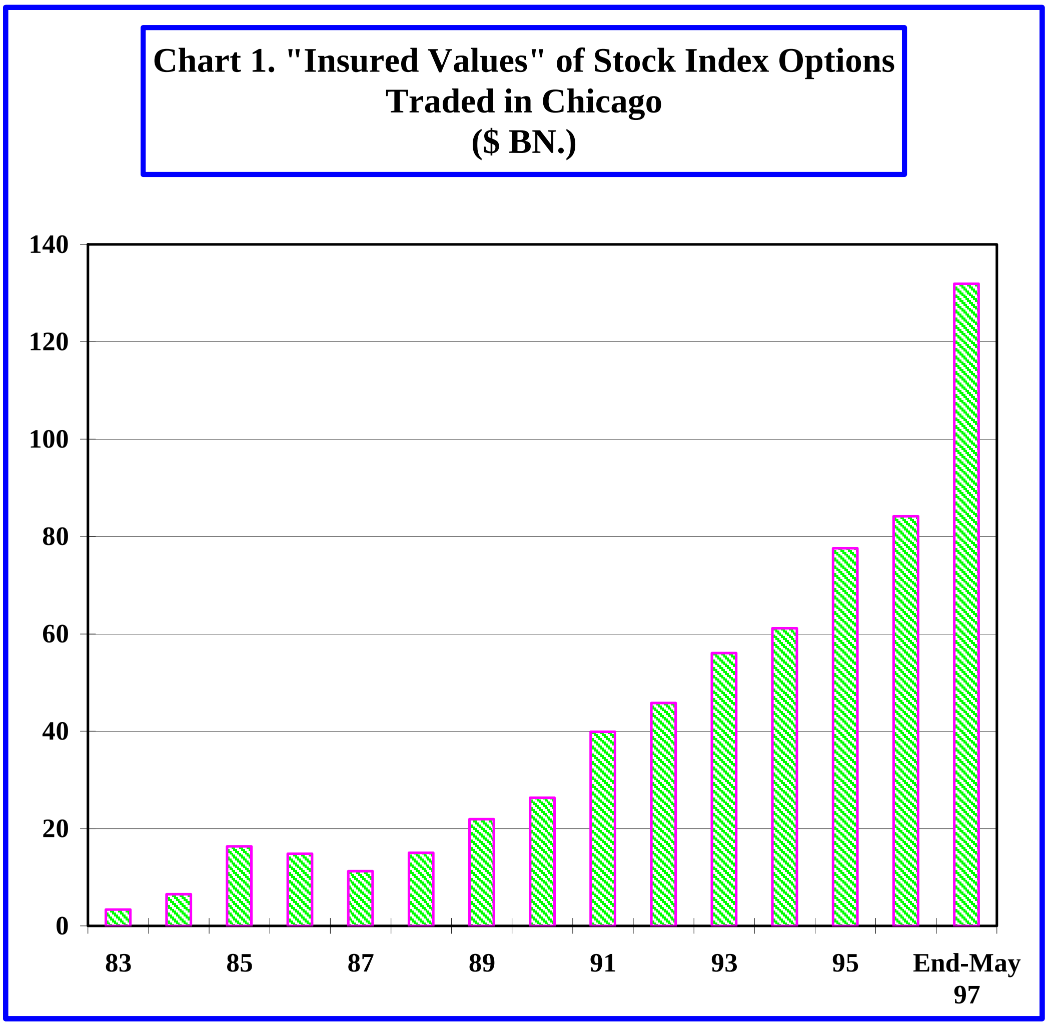
3. The Size of the Stock Options Market .

As risk rises with the amount of insurance being provided by the options market, it is important to be able to measure the size of the exposure. Unfortunately the answer is far from simple. For traded options,

[1] A put option allows the owner to sell the market at a given exercise price, which is usually below the current market level. If the price at which the option can be exercised is equal to the current price of the underlying asset, then the option is said to be "at the money", if above "in the money" and if below "out of the money".

[2] Since the emphasis in this report is on the insurance rôle of options markets, we discuss the issues in terms of put options. The principles apply equally to call options, which can be used for the same purpose. For example, an investor in the stock market who wishes to purchase full protection against market falls could do so either by holding the market and buying an appropriate number of at the money puts, or by selling the market, holding T-Bills, and buying an appropriate number of at the money calls. “Put-Call Parity” ensures that either of these approaches should provide the same amount of cover, at the same price.

information is widely available on the volume of trade and for the "open interest", which is the number of contracts outstanding. These numbers do not, however, readily translate into money values and for the over-the-counter market even these figures are not widely available.



The ideal measure of the size of the options market would be an indication of the "insured value" which it provides, analogous to the value of the cover provided by any other insurance market. Chart 1 provides a rough indication of the total cover provided by one important (but relatively small) sector of the market, which is the face value of all index put options traded on the Chicago Board Options Exchange.[3]

[3] The measure of cover is derived by multiplying the number of outstanding put contracts by the dollar value of each contract. Since the number of outstanding puts is roughly equal to the number of calls this is approximately equal to half of the notional value of the entire options market. The caveat in the previous footnote should be borne in mind, since in principle some proportion of outstanding calls may also represent a form of insurance.

The rapid growth in the market is illustrated by the fact that the "insured value" has increased over twelvefold in the past decade; even over the past 2½ years the insured value has more than doubled.

The amount shown in the chart, however, represents only a fraction of the total options market. Table 1 gives an estimate of the possible order of magnitude of the insured value represented by all stock options, including other traded index options, individual stock options, and the over-the-counter market. The figures quoted must be viewed with caution, but they suggest that the proportion of the US equity market which is "insured" via the options market is approaching 10%.

Table 1. "Insured Values" from Stock Options as of end-May 1997.[4]

Chicago Board Index Options	$132 bn.
Total US Index Options	c. $200 bn.
Total Traded US Equity Options	c. $300 -$400 bn.
Over-the-Counter US Equity Options	c. $200 - $300 bn.
Total US Equity Options	$500 - $700 bn.
(As % of Total US Equity Market)	7% - 9%
Estimated Total, All Major Markets	*c. $1,400 - 2,000 bn.*

[4] See Appendix 1 for comment on the derivation and uncertainty regarding these figures. Note that all figures in this report should be regarded as illustrative, rather than precise.

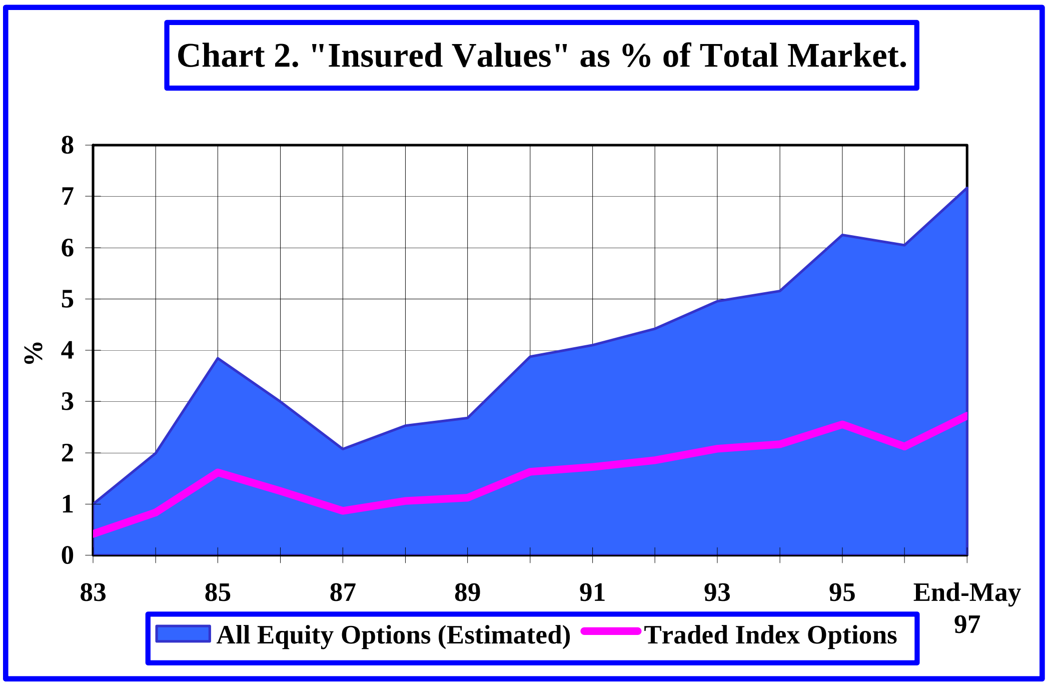


Chart 2 shows estimates of insured values as a percentage of the total value of the US equity market. The chart makes it clear that, although the precise extent of the cover may be uncertain, the rôle of options in insuring against market losses appears to have grown very significantly in recent years.[5]

For small falls in the index, most put contracts will not be exercised, but the proportion will rise with the size of the fall. A fall of 30-40% in the index would mean that almost all put contracts outstanding would be exercised, since exercise prices on puts are rarely more than this percentage below the current index level. Hence the larger the fall in the index envisaged, the better is the series shown in the chart as a measure of the true cover provided by stock options.

The figures in Table 1 suggest that, were a major fall in the index to occur, large sums of money would be paid out. A fall of, say, 30% in the market from current levels would probably imply an average payout of at least 20% of the insured value shown.[6] Table 2 quantifies the likely size of

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[5]Note that the figures in the chart correspond to the lower end of the ranges shown in Table 1.

[6] The payout on any individual contract is equal to the difference between the current price at the time the option is exercised and the exercise price, not the fall in the index itself. The 20% figure assumes that the average exercise price is 10% below the current price. Since the average exercise price is not affected by the magnitude of the fall, the cost of "insurance claims" will rise more than proportionately with the size of the market's fall.

this payout. The final line is calculated on the assumption that any major fall on Wall Street would be accompanied by a similar fall in other major markets.

Table 2. Estimated Payouts from Options Contracts.	
Market Capitalisation of US Equities	$8 trn.
Value of "Insured" portion	$500 - $700 bn.
Estimated Payout on 30% price fall[7]	$100 - $140 bn.
Payout on all major markets	*$280 -$400 bn.*

4. The Risks to Option Dealers.

Although in some respects the options market resembles any other insurance market, in others it differs markedly. The insured values shown in Table 1 are, for example, much smaller than the insured value of the US housing stock,[8] but the insurance industry can carry such massive contingent liabilities because the risk is specific rather than systemic. The specific risk is the risk of one house burning down, the systemic risk is the risk that all houses will burn down at the same time. The problem with the options market is that share price movements are highly correlated and this is increasingly becoming true of markets world wide. Thus while it is unimaginable that 89% of the US housing stock would suddenly be wiped out, the equivalent disaster happened in the US stock market between 1929 and 1932.

Systemic shocks, as for example a major nuclear disaster, are commonly regarded as uninsurable. Implicitly, it is assumed that there is, however, one ultimate underwriter which would take over in such a situation, namely the taxpayer. This parallel is ominous for the options industry and suggests the probability of moral hazard influencing the capital backing of option dealers.[9]

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[7]After “out of the money” allowance.

[8] The Federal Reserve’s *Balance Sheets for the US Economy* gives the value of the housing stock which is of similar magnitude to the value of quoted equities. We presume, however, that the proportion of the housing stock that is insured is much greater than the proportion of the stock market.

[9] Those buying insurance through the options market will tend to assume that the major financial institutions with whom they deal are "too big to fail". They will therefore be willing to do business with those institutions without being

The stock options market differs significantly from most other insurance markets because almost all of the risks being insured are systemic, rather than specific. If our estimates are broadly correct, the ultimate underwriters of stock market insurance are at risk to the extent of \$300 to \$400 bn. in the event of a 30% fall in the major stock markets. The dealers in options have, however, neither the wish nor the capital to accept risks of this magnitude. For this reason they normally hedge their exposure to the market's directional movements, which is known as “delta” exposure. But two other types of risk known as “gamma” and “vega” exposures, whilst in principle hedgeable for any individual dealer, cannot be hedged by the dealer community collectively.

The growth of the options market thus involves financial markets becoming increasingly risky, in a way analogous to the increased risk to an economy which is involved with the growth of debt.

5. Delta Hedging.

The “delta hedge” underlies the whole principle of options pricing. Delta hedging neutralises the impact of changes in stock prices, in either direction, provided the price changes are small.

The simplest example of delta hedging is usually presented in terms of transactions involving a single option and the underlying stock.[10] If the price of a given stock changes, the price of the option will also change, by an amount usually represented by the Greek letter Δ (“delta”). The relationship between the change in the option price and the change in the stock price is stable for small fluctuations. The sellers of the option can thus hedge themselves against loss, by taking the opposite position in the stock to the risk they have assumed on the option i.e. if they will lose money on a call option if the stock price rises, then they buy the amount of the stock which gives them an equal profit. This amount must equal Δ times the exposure on the option - hence the term “delta hedging”.

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overly concerned as to whether they have adequate capital to pay out in the event of a crash. Competitive pressures will then make it difficult for major financial institutions who write options, to be conservative in their estimates of the capital required to back their dealing positions.

[10] The underlying stock may of course be an index, which cannot itself be bought and sold, but a futures contract thereon can be, and the argument therefore goes through with only minor modification.

The link between insurance and speculation becomes evident by following this relationship through. Buyers of puts wish to reduce exposure to the market. Since the delta hedging traders wish to have zero exposure to the market, they must thus find others who wish to increase exposure.[11]

Crucially, a trader who sells both calls and puts will tend to hedge delta exposure automatically, thus reducing the need to engage in transactions in the underlying stock. The following example shows how this works. On the 10th July, one month put and call options on the S&P 500 with an exercise price of 920 had almost exactly the same delta, with opposite sign, and almost exactly the same price ($20 and $22 respectively). A dealer selling a thousand of each would have had liabilities of around $4 million[12], and would be more or less "perfectly" delta hedged. If the market had fallen by 1%, the price of the put would have risen by around $5, whilst the price of the call would have fallen by around $4, leaving total liabilities little changed.

6. Gamma Hedging.

Delta hedging cannot, however, cover all the risks underlying the options market. Unlike the risks, which for example a bookmaker undertakes, the payoff to a purchaser of an option is determined by the scale of the change in the price of the underlying stock, and, crucially, in a non-linear way. It is as if the bookmaker paid out more on the winning horse depending on the square of the winning distance. For this reason delta hedging only protects the option dealer against the impact of relatively small changes in prices.

In the example given above, the dealer had a near-perfect delta hedge against small movements in the market, but in the case of larger movements, the situation would have been very different indeed. If, for example, the market had fallen by 10%, the call would have become essentially worthless, whilst the put would be worth around $90 - roughly the difference between the exercise price and the new stock price. The liabilities of the option dealer would therefore have increased from $4 million to $9 million.[13]

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[11] The way in which the market operates the delta hedge is shown in more detail in Appendix 2.

[12] Since the contracts are on 100 times the index.

[13] This example was calculated using the industry-standard Black-Scholes pricing model and assuming no change in underlying volatilities. Most practitioners use more sophisticated models, but nonetheless these give similar results in the face of such a large shock. Whilst the example was given in terms of

This is an example of "gamma exposure".[14] If prices move reasonably smoothly, this type of exposure can be limited by "rebalancing", which requires progressively changing the degree of delta hedging as delta itself changes. Whilst gamma exposure can be reduced in this way, it can never be entirely eliminated. This is probably the most crucial respect in which the practice of options dealing differs from the textbook case which underpins the standard "Black-Scholes" pricing model. In this model rebalancing is assumed to be possible on a continuous basis, which implies that writing options and delta hedging is a riskless activity; in practice it can never be.

It is of course possible to hedge not only against delta exposure, but also against gamma exposure. But there is a very crucial difference between delta hedging and gamma hedging. As noted previously, the activity of writing options has a tendency to self-hedging against delta risk, thus systemic price risk is underwritten semi-automatically and in advance. But whilst gamma hedging is possible for any single dealer it is impossible for the financial sector as a whole. The only way to hedge gamma exposure is to buy, as well as sell options. But this simply moves the gamma exposure from one dealer to another. The financial community as a whole cannot avoid gamma exposure and in practice this risk will be largely borne by option dealers.[15]

As the gamma exposure must be largely borne by the option dealers its size is important. As with all other aspects of options, this is very hard to quantify. Table 3 gives an estimate of aggregate gamma exposure for one small segment of the market to a price shock, which is assumed to occur sufficiently rapidly to rule out rebalancing.

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a falling market, it is central to the definition of gamma risk that it applies equally to sharp rises as to sharp falls.

[14] Mathematically, the delta of a portfolio, Δ, is given by $\partial\Pi/\partial S$, where S is the underlying stock price; gamma is given by $\partial^2\Pi/\partial S^2$. The gamma of the portfolio in the example is larger than in the standard textbook hedged portfolio - where the delta hedging is carried out with the underlying stock rather than another option - because both calls and puts have a positive gamma, whilst the stock's gamma is zero.

[15]The only way the dealer community as a whole could avoid gamma exposure would be if long term investors had equal exposure to both put and call options, as both buyers and sellers. This would run against the natural activities of long term investors and it raises problems of credit exposure. Indeed, if such a situation were to occur, the primary *raison d'être* of options dealers - to act as intermediaries who remove credit risk - would disappear. This issue is discussed further in Appendix 3.

Table 3. Gamma exposure shown by the impact of a 10% price shock on market in short-dated options on the S&P 500 traded at the Chicago Board Options Exchange.[16]	
Face Value of Market	$152.7 bn.
Market Value of Market	$1.62 bn.
Gamma Exposure to 10% Shock	$-3.37 bn.
(as % of Face Value)	- 2.2%

Translating this to a figure for all equity options, comparable to the figures in Table 1, cannot be carried out at all precisely. The gamma exposure of the market as a whole is almost certainly significantly lower (in relation to the size of the market as a whole)[17], but it seems reasonable to assume that the exposure of the entire market to a rapid 10% price shock would be something of the order of 1% of the face value of all options or around 2% of the "insured values" shown in Table 1 (assuming roughly equal numbers of puts and calls). This would imply aggregate losses to option dealers of something between $10 and $15 bn. on the US market, and up to $40 bn. worldwide, if other markets fell too. It should also be noted that whilst this shock would be a very severe short-term market correction, it is less than one half of the fall on 19th October 1987. Furthermore the scale of the gamma exposure does not increase proportionately with the size of the shock, but rather with its square. Hence doubling the shock would increase the gamma exposure by four.

Gamma exposure itself only arises in response to rapid price movements. If price falls were slower, gamma risk would be reduced, but only at the expense of severe selling pressure on underlying stocks and in the futures market, as institutions who had written options struggled to

[16] As of 10 July 1997. The calculation is an approximation using an estimate of the aggregate (Black-Scholes) gamma of the market derived from individual gammas on all options traded on that day. Inevitably, it also ignores double counting; although this probably does little to change the relative magnitude of the gamma exposure to the size of the market.

[17] Short-dated options have markedly higher gammas than do long-dated. Taking all other factors as given, an option with a maturity of one month, which is roughly the average maturity of those represented by Table 3, has a gamma roughly 3 ½ times that of one with a maturity of a year. We assume that the average gamma for the market as a whole is only half that for the sector of the market in Table 3.

rebalance their portfolios. This must inevitably imply an accentuation of any fall once it starts in earnest.

There is an obvious parallel with the role of portfolio insurance during the 1987 crash. One of the morals of the crash was that put options were preferable to portfolio insurance, which could not operate when the market moved rapidly. But the shift from portfolio insurance to the options market, which is indirectly visible in Chart 2, simply shifts the problem on, it does not eliminate it. Before the 1987 crash, portfolio insurers effectively bore their own gamma risk, now they have passed this risk on to options dealers.

Another claim often made about the 1987 crash was that the actions of portfolio insurers accentuated the severity of the fall, since they were obliged to sell into the falling market.[18] If this is the case, the same must be true for the options dealers who have taken over from the portfolio insurers. Inevitably, as the size of the options market grows relative to the underlying market, the impact of this behaviour on market movements must increase. Thus it seems highly likely that the risk of sharp market movements (or "discontinuities") increases as the options market grows.

7. Vega Exposure.

An additional risk which options dealers collectively cannot hedge away is their collective exposure to the market's own estimate of volatility, which is usually referred to as vega exposure.[19] For the same market illustrated in Table 3, an increase of five points in assumed volatility, which is roughly comparable to the increase seen in the immediate aftermath of the 1987 crash, would increase CBOE option dealers's collective liabilities by around \$0.5 bn., or around one third of a percent of face value. Total exposure for the market as a whole would be larger, in relation to face value, but would probably not match total gamma exposure.[20]

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[18] It should be noted that whilst this assertion must logically have some foundation, there is considerable controversy as to the quantitative significance of portfolio insurance in the Crash.

[19] Mathematically, vega is given by $\partial\Pi/\partial\sigma$, where σ is the volatility assumed in pricing the option. Since both call and put options have a positive vega, this risk is not hedgeable collectively. Strictly speaking, the idea of vega exposure contradicts the standard Black-Scholes model which assumes that σ is a constant, but more sophisticated models of stochastic volatility have similar properties, under reasonable assumptions.

[20] Vega, in contrast to gamma, increases with the maturity of the option. An option with a maturity of a year has a value of vega around 3 ½ times larger than the equivalent option with a maturity of one month.

While vega exposure is quantitatively less significant than gamma exposure, it is important because changes in gamma and vega can and almost certainly will be reinforcing. If the market suddenly falls 10%, the gamma exposure will give rise to large losses. It is likely, however, that the market's estimate of vega will now increase and this will give rise to additional losses from the vega exposure.[21]

Table 4. Estimated Exposures of Investors and Option Dealers to a Sudden Fall on Wall Street.[22]

	Investors' Exposure (Delta)	US Dealers' Exposure (Gamma and Vega)	Global Dealer's Exposure
10% Fall	$800 bn.	$15 - $20 bn.	$40 - $55 bn.
20% Fall	$1,600 bn.	$55 - $75 bn.	$150 - $210 bn.

Because of the difficulty of hedging the gamma and vega exposures the options market is liable to experience major losses in the event of a sharp fall in the stock market. This loss is likely to rise in a non-linear way with the extent of the fall.

[21] This is indeed assumed in many pricing models, which assume volatility to be negatively related to the share price. In practice, as we show below, option prices have been almost solely determined by recent changes in actual market volatility. It is therefore almost certain that vega exposure will rise in the event of a rise in volatility.

[22] Estimating the combined gamma and vega exposure of options on the US equity market involves heroic assumptions. Table 4 assumes that all the delta exposure is taken by long term investors, with regard to both the "insured" and "uninsured" portions, (implying that dealers collectively are perfectly delta-hedged). All of the gamma and vega exposures are assumed to be taken by the option dealers. The vega exposure is assumed to be around one third of the gamma exposure in the case of a 10% fall. The global figures assume other markets fall in line with the US.

8. The Natural Imbalance in Long Term Investors' Option Positions.

A central thesis of this Report is that the gamma and vega risks are borne, to an important extent, by option dealers rather than long term investors. If the option market was not fundamentally different from the stock market, then option dealers would simply match buyers and sellers and would be left with no residual exposure. The reason why the option market differs from the stock market in this respect arises from two features. The first is the "Put-Call" parity and the second is the natural bias of long term investors. If the markets in puts and calls were entirely separate, then the prices of puts and calls would frequently diverge. That they are usually in balance, is due to arbitrage by option dealers. While this arbitrage can be achieved without the option dealers being at risk with regard to the direction in which the market will next move, it cannot be carried out without exposing dealers to the two types of volatility risk, gamma and vega.[23]

9. Credit Risk.

As in stock markets, long term investors do not deal directly with one another. In both instances this places the short term credit risk of investor default on the dealers. In option markets, however, there is no title to an asset which can be transferred from one client to another, so the market's credit risk lasts until the option period expires. The consequence of this is that a market shock, which is sufficient to cause significant bankruptcies among clients or dealers is liable to have a knock-on effect. If a dealer finds that, through default, his position has become unexpectedly uncovered, he will seek to "recover" it. This will, in turn, tend to accentuate and accelerate the directional change of the market and this will tend to increase gamma and vega exposures.

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[23] That an arbitrage element is needed, is shown by the fact that put-call parity does not always hold. In Appendix 3 we seek to illustrate why the natural stance of long term investors gives rise to the opportunities for the arbitrage which is needed to achieve put-call parity. This arbitrage is not riskless. Speculators may also, of course, take positions, but they will tend to replace long term investors rather than option dealers due to credit risk as well as natural inclination.

10. Rising Options Prices and Market Overvaluation.

We have shown in a number of past reports that there is strong evidence that the US stock market is wildly overvalued, indeed at current levels significantly more than in August 1929. In the face of such a major distortion, it is of no inconsiderable interest to investigate whether options which derive their value from the US equity market are also mispriced.

An obvious issue to investigate is whether options prices show any reflection of this overvaluation. It is to be assumed that were a major hurricane heading towards the Southern States of the USA, premia on buildings insurance would be on an upward trend. It is thus of interest whether or not the same is true for the insurance provided by the options market.

At a superficial level, it might appear so. Since the start of 1996 options prices and their "Implied Volatilities", have risen sharply. Implied volatilities can be interpreted as the options market's implicit forecast of the variability of the stock market itself.[24] Accordingly, the rise in these implicit volatilities might be viewed as revealing a growing awareness of market overvaluation.

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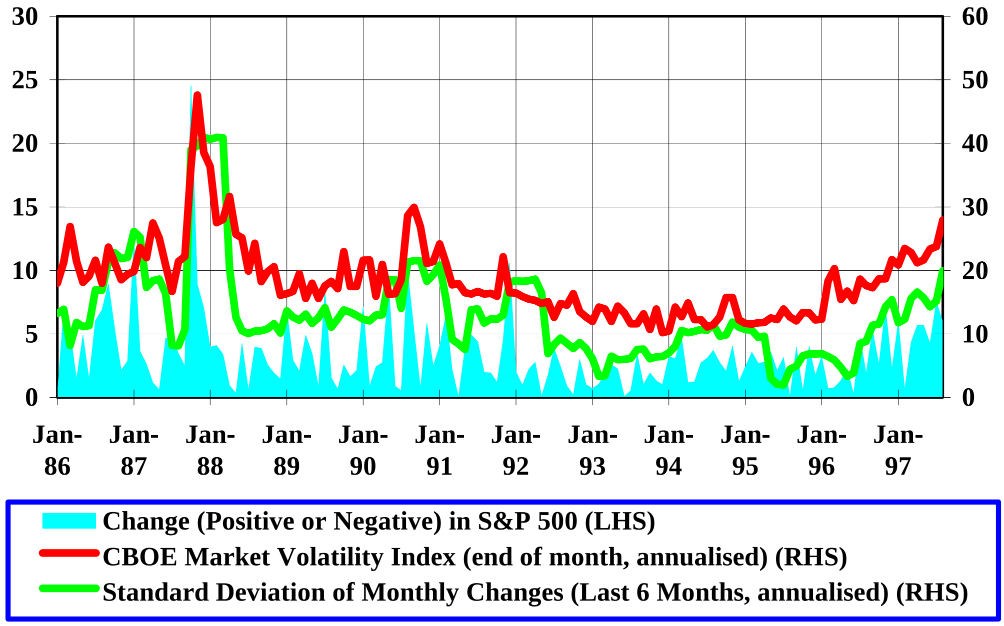
[24] Implied volatility is calculated by inverting the equation $p(\sigma, \mathbf{X}) = P$, to find σ, where $p(\ .\ )$ is the price as a function of σ and $\mathbf{X}$, a vector of other observable factors, from a given pricing model (usually Black-Scholes) and P is the observed options price. Thus, strictly speaking this statement is only correct if the market is using the given pricing model used in deriving the implied volatility. The well-documented fact that Black-Scholes implied volatilities vary significantly as a function of, e.g., strike price and volatility, makes this assumption questionable. But since all options prices are increasing in volatility, another way of viewing implied volatility is simply as a transformation of options prices to a more readily interpretable and comparable form.

Chart 3. Actual and Implied Volatility 1986 - 1997.



However, as Chart 3 shows, this inference would be incorrect. The chart shows a measure of implied volatility calculated by the Chicago Board of Options Exchange, alongside two indicators of actual stock market volatility. The first is a fairly standard measure, being the standard deviation of market price changes over the past six months, expressed at an annual rate.[25] The second is a more direct indicator of market volatility, being the absolute value of actual monthly price changes (i.e. with both negative and price changes shown as a positive number).

The rolling standard deviation responds by its nature, to the absolute value of the actual changes with a lag. More notably, however, the indicator of implied volatility responds in more or less the same way. This implies that whilst implied volatility should be based on risk assessment, allowing for both the historic range of fluctuations and the relative size of the options market, in practice it is based on short term fluctuations, simply responding to shocks that have recently hit the market, rather than anticipating the likelihood of future problems. The collective memory of the options market appears to be very short-term in nature, with implied volatility simply responding to the events of the past six months.

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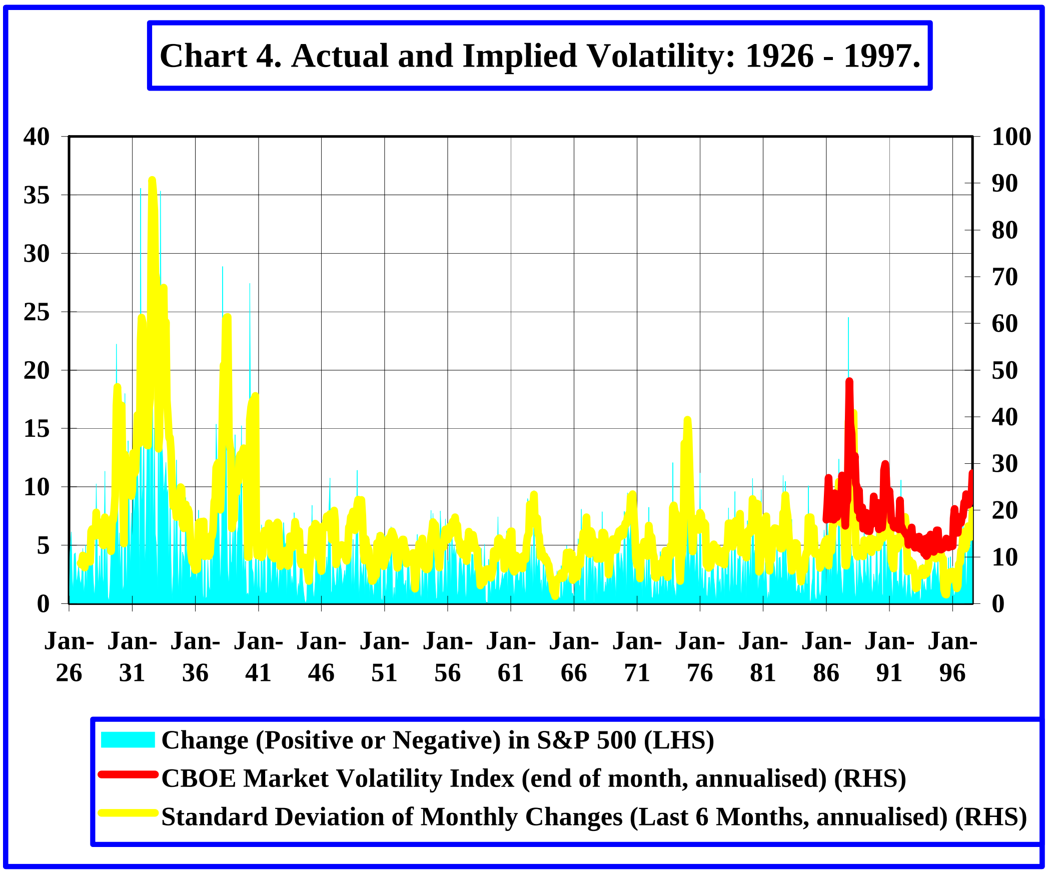
[25] The figures in the chart are formulated on month-on-month price changes, rather than day-by-day changes, as used by market practitioners. This is one of the penalties of using longer historical databases - but in practice does little to affect the figures.

These features are evident simply from an inspection of the chart, but can be substantiated by more quantitative tests.[26]

One way to put Chart 3 into perspective is to lengthen the sample to include more observations of market shocks. Although it includes the 1987 crash, this is the only major "event". Chart 4 goes back 70 rather than ten years. (Unfortunately this is only possible for actual volatility figures - implied volatility figures of any kind are very hard to obtain and appear unobtainable before the mid-80s.)



The longer sample shows that, whilst actual volatility has been rising of late, it is still below many historic peaks. Notably and unsurprisingly, many of these followed periods of major overvaluation.

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[26] Appendix 4 shows that implied volatility in any month can be predicted with some precision by an econometric equation driven by the absolute value of actual shocks that have hit the market over the past year, with most of the predictive power coming from the most recent six months. The appendix also shows that implied volatility is, effectively, of no use at all in predicting actual volatility.

Table 5. Actual and Implied Volatility, compared to Historic Averages.			
	Actual	**Implied**	**Premium**
End-August 1997	19.9	27.9	8.0
1986-1997	15.0	18.3	3.3
1988-1997	11.7	17.1	5.5
1926 - 1997	19.7	N/A	N/A

Table 5 summarises the relationship between actual and implied volatility. It shows that actual volatility is currently significantly above its average level over the past eleven years.[27] The premium of implied volatility over actual is also significantly above its average over the same period. However, since this sample includes a major prediction error by failing to spot the 1987 crash, it is probably more appropriate to look at the average premium since 1988, shown in the third row. On this basis the premium looks high, but less markedly so. Chart 3 shows that the premium appears to have been fairly stable for the past two or three years - the major development has been an increase in actual volatility, not in the premium that option dealers charge. By implication, if actual volatility drops back, so will options prices - even if the risk of a major collapse does not go away.

If history only consisted of the past eleven years and if there were nothing unusual about the current state of the US equity market, this would suggest that options were fairly valued, albeit unintentionally, given the short term nature of the way in which implied volatility responds to market events. But the final row of the table is a reminder that, in comparison with a longer run of historical data, recent market volatility is still only around its historic average.

[27] "Actual volatility" can only be proxied by taking a short-term rolling average - in this case the rolling standard deviation over the past six months. Volatilities are calculated from month-on-month log changes in price and expressed at an annual percentage rate.

11. Conclusions.

- The options market provides insurance to long term investors.

- This insurance can be obtained for fully invested portfolios through buying puts and for liquid investors through buying calls. The ability to achieve the same result by two different routes means that there is a natural arbitrage between the prices of puts and calls.

- Investors can also use the options market to take speculative positions.

- If long term investors were equally prone to buy and sell puts and buy and sell calls, then option dealers would not find opportunities to arbitrage the put-call parity. In practice this arbitrage is one source of their profitability and it involves risk.

- The risks to which option dealers are exposed are systemic. Whether or not options are correctly priced dealers will usually find the business profitable, but on rare but dramatic occasions it will be spectacularly unprofitable. This type of risk habitually gives rise to "catastrophe myopia", which leads to underpricing and undercapitalisation. This is also encouraged by moral hazard if the state is regarded as the lender of last resort.

- The risks to the stock market and economy are considerable. First because the growth of the options market increases the likely size of price discontinuities and thus of self-reinforcing price spirals. Second because the undercapitalisation of option dealers increases the risks of bankruptcy among dealers, which will in turn tend to reinforce the problems created by a stock market crash.

- It is possible, but not very likely that the growth of the options market could set off a crash, through the impact on price discontinuity.

- The greater risk is simply that it will add to the severity of a crash set off by some other factor.

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Appendix 1.

The Source and Reliability of Data on Options.

In this report we refer to the problem of obtaining accurate data on the options market and to the consequence that the figures that we produce should be used carefully to indicate orders of magnitude rather than precise numbers.

Our estimates of “insured values”and the impact of stock market falls are set out in Tables 1,2 and 3. The most reliable statistic is the first in Table 1, which is taken from data published by the Chicago Board Options Exchange (CBOE), as insured values can be calculated fairly straightforwardly for index options, and the CBOE provide totals for both put and call options. However, even this figure is open to doubt, since it will include an element of double-counting due to inter-dealer transactions.

Subsequent figures in Table 1 become increasingly unreliable as the table proceeds. The second row is derived from BIS data, published in *International Banking and Financial Market Developments* for notional amounts outstanding for all traded US Equity Index Options, scaled by the proportion of puts in the CBOE data.

The third row, showing a likely range for all traded equity put options, includes an estimate of the “insured values” provided by the market for traded options on individual equities. We have been unable to obtain estimates of notional amounts on such contracts (the BIS only publish figures by number of contracts outstanding): the lower figure in the range shown assumes the total size of this sector of the market to be one half that of the index options market; the upper assumes the two sectors of the market to be of equal size.[28]

The lower of the two figures quoted for the over-the -counter market is based on an estimate of total notional amounts outstanding for March 1995 published in the BIS’s *Central Bank Survey of Foreign Exchange and Derivatives Market Activity.* The figure is again scaled by the proportion of puts in the CBOE data, and scaled-up on the assumption that traded and OTC put options have grown in line since then. The upper of the two figures is entirely unsubstantiated, but is based on estimates of relative magnitudes of the two markets given by a number of practitioners and regulators during private conversations, and allows for the possibility that

[28]In terms of numbers of contracts, the BIS data (published in *International Banking and Financial Market Developments*) shows the single equity contract market to be over five times larger than the index options market, but individual stock options are generally significantly cheaper on a per unit basis.

the OTC market may have been gaining in size relative to the traded options market since the BIS survey.

The final figure, giving an estimate for all major markets, is based on the relative size of the US options market, to the global total. For traded options, the global total was around 1.5 times the US in the latest data (for March 1997, in *International Banking and Financial Market Developments)*; for the OTC market, however, the global total was some 4.6 times the US (for March 1995, from the *Central Bank Survey of Foreign Exchange and Derivatives Market Activity*).

Chart 2, showing “insured values” as % of the total equity market, relies on CBOE figures as the primary source of information on trends over time (although trends in BIS traded options figures are very similar). BIS figures on individual equity option contract numbers suggest that this sector of the market has grown especially rapidly in the past three years - thus we assume it to have represented a smaller sector of the market in earlier years. There is no time series information on the size of the OTC market, hence of necessity this is projected backwards on the basis of growth rates in the traded sector.

Appendix 2.

Delta Hedging.

If, when the price of the underlying stock changes by $1, a given option price changes by an amount $\$\Delta$ (“delta” - which may be positive or negative), a portfolio made up of -1 times the option, (i.e. selling the option, hence holding it as a liability) and Δ times the stock, or index, will be unaffected by small changes in the market price, since the value of the option to the trader will change by $-\Delta$ and the value of the stock held will change by $+\Delta$.

For a call option Δ will be a positive fraction, hence a seller of a call needs to hold Δ times the underlying stock, or a futures contract thereon, to delta hedge. For a put option Δ will be negative, since the value of a put option rises as the market falls, hence a seller of a put needs to sell $|\Delta|$ (the absolute value of Δ) times the market in order to delta hedge. The simplest way to do this is to sell a futures contract of the appropriate size and maturity for the option. Table A1 illustrates a sequence of trades which cancel out for the market as a whole. As illustrated, the buyer of insurance and the speculator operate only in the options market, while the trader operates in both options and futures markets.

The example makes it clear that the speculator is absolutely crucial to the process of providing insurance, indeed the option trader, in the above example, simply acts as an intermediary in terms of "delta" (ie, essentially, directional) exposure.

Table A1. The Link Between Insurance and Speculation in the Options Market.		
Option Buyers.	**Option Trader who Delta Hedges.**	
Option Market		**Futures Market**
Risk-Averse Individual Buys Put Option	**Sells Put Option**	**Sells Futures Contract on Underlying Stock (or Index)**
Speculator Buys Call Option	**Sells Call Option**	**Buys Futures Contract on Underlying Stock (or Index)**

The table also brings out another crucial aspect of the options market. A dealer who is selling both put and call options is both a buyer and seller in the futures market. For some values of the call option and put option, a given trader's transactions in the futures market could in principle net out to zero. Thus a trader who sold one put option, and $|\Delta_P|/\Delta_C$ call options (where $|\Delta_P|$ is the absolute value of the put option's delta, and Δ_C is the call option's delta) would have zero exposure to the underlying stock price, since the change in the value of the trader's portfolio in response to a change of $1 in the stock price would be:

$$- \Delta_P - (|\Delta_P|/\Delta_C)\, \Delta_C = - \Delta_P + \Delta_P = 0.$$

In general such precise netting out is unlikely; but it does imply that a trader who sells both calls and puts automatically engages in at least partial delta hedging.

More generally, in a crucial sense, the activity of writing options is to a great extent self-hedging against delta exposure - only the net position, in effect, needs to be hedged. For the dealer community as a whole, the aggregate net delta exposure is likely to be small.

In this respect, as noted above, the options market can be viewed as dealing with the potential problem of systemic delta exposure by "up-front-underwriting". As noted previously, the ultimate providers of "delta" insurance, protection against market falls, are speculators. But since provision of such insurance by a more normal insurance mechanism would imply major credit risk, the cover provided by the buyer of the call is provided up-front, when the call is purchased.

An alternative analogy for the options market is the bookmaking industry. The gambler who picks the winning team in a football match is effectively paid by the gambler who picks the losing team. Provided the bookmaker has calculated his odds correctly, he has zero net exposure to the outcome of the match.

Appendix 3.

The Need for Option Dealers to Arbitrage Put-Call Parity.

A2. The Natural Options Positions of Long Term Investors.			
	Credit Cover	**Purpose**	
		Bear	**Bull**
Buy Calls	**None needed**	**None**	**Speculation**
Sell Calls	**Equity Holdings**	**Premium to fund insurance**	**None**
Buy Puts	**None needed**	**Insurance against fall.**	**None**
Sell Puts	**Cash**	**None**	**Premium to fund speculation**

Table A2 seeks to show that the activities of long term investors will not tend to produce a situation whereby the prices of call options will not naturally be in balance with the prices of put options. The asymmetry arises due to credit risk as well as the preferences of long-term investors.

A long-term investor holding the market, but nervous about falls, can insure against market losses by buying puts. In principle, this can be funded by writing calls, since the underlying stock holdings provide credit cover. This combination implies an asymmetric outcome in response to price falls vs price rises, but does not imply taking on any gamma or vega exposure (since exposure from writing the call will be offset by “negative” exposure from buying the put).

The supply of puts from long-term investors is however less likely to be forthcoming, since this must require credit cover in the form of cash. For this reason, in the absence of dealers, it might be expected calls would be relatively “cheap” compared to puts (i.e. put-call parity would not hold). Since this would in turn imply an arbitrage opportunity, it provides one rationale for the existence of the dealer community. But, crucially, this arbitrage is not riskless, since being a “net writer” of options implies taking on gamma and vega exposure.

Thus, without the arbitrage activities of dealers, the Put-Call parity would not normally be realised. Its realisation is not, however, a riskless activity and the necessary risks undertaken by option traders are those associated with the market's volatility rather than its direction.

Appendix 4.

Predicting Implied Volatility.

Table A4 shows the results of a regression of the CBOE’s estimate of implied volatility on lagged absolute price changes[29] over the previous year. The table shows that this produces a reasonable fit, with the majority of lagged price changes statistically significantly different from zero, but with most of the power coming from the most recent months.

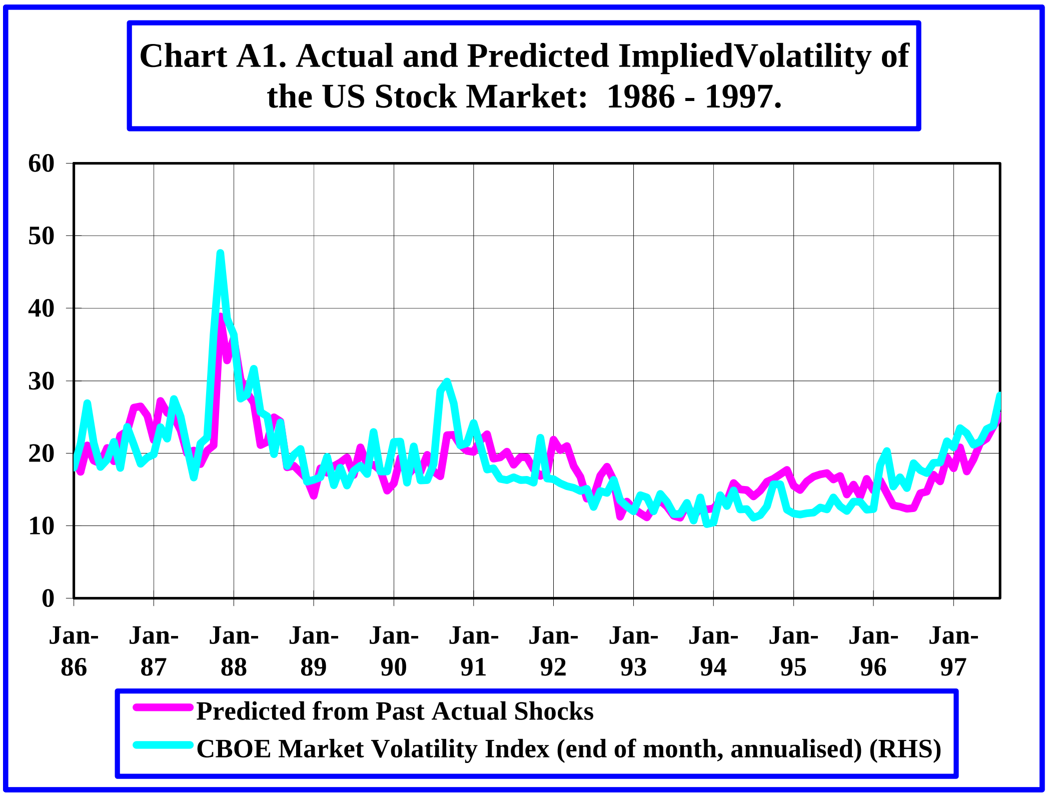
The sum of the coefficients equals 3.63, which is fairly close to expectations, since implied volatility is measured at an annual rate, but the absolute price changes are measured at a monthly rate. The average absolute monthly price change will approximately equal σ_M, where σ_M^2 is the monthly variance. If the share price is close to a random walk, then the annual variance, σ_A^2 will equal 12 σ_M^2, and hence we should find $\sigma_A = 12^{0.5} \sigma_M$, and $12^{0.5} = 3.46$. The intercept of around 6.5 can be interpreted as the average option premium.

Defined as $|dp| = |\Delta \ln P * 100|$, where P is the S&P 500. Scaling the log change by 100 transforms it, to a close approximation, to a percentage change, but without the bias in percentage changes that a given percentage fall, followed by the same percentage rise, does not restore the index to its previous level.

Table A3. Results of Regression of Implied Volatility on Lagged Absolute % Price Changes (|dp|)			
	Coefficients	Standard Error	"t"- Statistic
Intercept	6.52	0.97	6.72
$\lvert dp\rvert_{t-1}$	0.83	0.12	7.19
$\lvert dp\rvert_{t-2}$	0.53	0.12	4.57
$\lvert dp\rvert_{t-3}$	0.60	0.11	5.28
$\lvert dp\rvert_{t-4}$	0.33	0.11	2.91
$\lvert dp\rvert_{t-5}$	0.23	0.12	1.96
$\lvert dp\rvert_{t-6}$	0.19	0.11	1.63
$\lvert dp\rvert_{t-7}$	0.09	0.11	0.75
$\lvert dp\rvert_{t-8}$	0.26	0.12	2.24
$\lvert dp\rvert_{t-9}$	0.32	0.11	2.81
$\lvert dp\rvert_{t-10}$	0.34	0.11	2.95
$\lvert dp\rvert_{t-11}$	-0.09	0.12	-0.76
$\lvert dp\rvert_{t-12}$	0.01	0.12	0.08
Adjusted R-Squared		0.59	
Standard Error		3.79	
Observations		137.00	

Chart A1 shows the performance of the equation in tracking changes in implied volatility. The equation exhibits some serial correlation, but this is to be expected, given the absence of lagged dependent variables in the regression. The chart shows that the recent upswing in implied volatility has been well-captured by the regression predictions.

These results suggest strongly (and unsurprisingly) that implied volatility is essentially backward-looking. In contrast, there is no evidence that implied volatility itself is of any value in predicting actual volatility. Running the regression in reverse (with lags of implied volatility used to predict actual volatility) gives a barely statistically significant adjusted R-Squared of 0.04, implying no more explanatory power than a pure autoregression.



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